ACTUARIAL MODEL OF THE IMPACT OF LINKING ECONOMIC VARIABLES TO A LIFE SURVIVAL FUNCTION

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Many Cuba observers have mentioned that there will be no transition to a market economy in Cuba until Cuba's current leadership passes away either physically or politically. This observation has served as an undercurrent in a variety of forums ranging from foreign policy position papers[1] and think tank publications[2] to more accessible publications such as novels,[3] magazines,[4] and even television commercials.[5] Missing in all these media have been a quantification of how long will the current leadership remain in place, and what is the likely result of waiting for its demise on the Cuban economy. This article will establish a framework for answering both of these questions by first defining a probabilistic survival model for the current leadership, and then using it to establish an actuarial model using demographic and economic assumptions.

Actuarial models are designed to produce estimates of the expected value of one or more economic variables that are distributed in a random manner. One such model assumes that the value of an economic variable is dependent on the survival or demise of individuals or objects. Examples of these are actuarial models that are used to determine present values of future payments contingent on the survival status of individuals such as pensions or life insurance benefits.

Making a particular economic variable dependent on the survival status of the current Cuban leadership, and making the leadership's survival status a random variable dependent on time elapsed, defines an actuarial model for that economic variable. Some economic variables that can be modeled in this manner are: the relative size of the Cuban economy, the expected value of expenditures such as economic aid or government outlays, and the average Cuban GDP per capita. The value of each of those variables is assumed to depend on the survival status of the current Cuban leadership.

THE SURVIVAL MODEL

The survival model is defined by the underlying random variable T where T is the time elapsed from an origin in time (usually birth) to the failure of an object or process, or the death of an organism. The probability of surviving during a particular time period can be defined as ${}_tp_x$ where t is the time elapsed since the attainment of age x. The corresponding

1. Council on Foreign Relations, Independent Task Force (Bernard W. Aronson and William D. Rogers, Task Force Co-chairs 1999), *U.S.-Cuban Relations in the 21st Century*. New York: Council on Foreign Relations, 1999.

2. Gonzalez, Edward. *Cuba's Dismal Post Castro Futures*. Santa Monica, California: RAND, 1996.

3. Cruz Smith, Martin. *Havana Bay*. New York: Random House, 1999; Patrick, Vincent. *Smoke Screen.* New York: William Morrow, 1999.

4. Putnam, John J. "Cuba: Evolution in the Revolution" *National Geographic Magazine* (June 1999).

5. Solomon Smith Barney. *What will happen in Cuba after Castro is gone ...?* TV Commercial aired in the United States, 1998-1999.

probability of not surviving a particular time period t is defined as ${}_tq_x$. These two probabilities are complementary:

$$ {}_tp_x + {}_tq_x = 1 \qquad (1) $$

Actuarial notation simplifies the above equation for a survival period of one year as follows:

$$ p_x + q_x = 1 \qquad (2) $$

The probability of surviving t years after attaining age x, and then not surviving the following year, can be defined as ${}_tp_xq_{x+t}$. This analysis will then use the discrete curtate survival random variable T_x with ${}_tp_x \cdot q_{x+t}$ as its probability mass function for some age x>0, and t=0,1,2,3,... The expected value of a function $f(T_x)$ can be defined as:

$$ E[f(T_x)] = \sum_{t=0}^{\infty} f(t)\ {}_tp_xq_{x+t} \qquad (3) $$

Similarly we can define the variance of $f(T_x)$ as:

$$ Var[f(T_x)] = \sum_{t=0}^{\infty} f(t)^2\ {}_tp_xq_{x+t} - (E[f(T_x)])^2 \qquad (4) $$

The expected value of $f(T_x = t) = t$ is defined as the curtate-expectation-of-life[6] or the expected curtate-future-lifetime. The expected curtate-future-lifetime can be equivalently calculated using the following simpler formula:[7]

$$ e_x = \sum_{t=0}^{\infty} {}_{t+1}P_x \qquad (5) $$

The expected curtate-future-lifetime at age x can be used as an estimate of the future lifetime of an individual who has attained age x. Death is assumed to occur on or before the attainment of the next future year of age. If the time of death is distributed uniformly throughout the year, the expected lifetime is known as the complete-expectation-of-life and it can be approximated by $e_x+½$.

One objective of the actuarial model is to estimate the expected value of a particular economic variable in the finite future. The value of that economic variable will be defined by a function $f(T_x,n)$ where T_x is the discrete random variable time to failure or death given a life already x years old, and n is the number of years elapsed from now (x). The expected value of $f(T_x,n)$ for $n<\infty$, is:

$$ E[f(T_x,n)] = \sum_{t=0}^{n-1} f(t,n)\ {}_tp_xq_{x+t} + f(\infty,n)\ {}_np_x \text{ for } n > 0 \qquad (6) $$

where $f(\infty,n)$ is the value of the economic variable n years in the future assuming that leadership has survived to that date.

THE ACTUARIAL MODEL

An actuarial model can be based on a discrete random variable survival model and a set of economic and probabilistic assumptions. The principal assumption is the probability survival function. This model can easily work with a probability function that defines the leadership survival in terms of quantifiable economic, political, or social variables. Nonetheless for illustrative purposes, it will be assumed that the leadership survival will be linked only to the physical survival of the leader or of members of the leadership. Furthermore it will be assumed that the leadership will survive as long as one member of the current or original leadership is alive.

The choice of an appropriate survival function can be debated ad nauseaum but for the illustrative purpose of this paper, two mortality tables previously used by the author will be assumed. The first mortality table has been derived by the author from Cuban published sources.[8] It has been previously used to project

6. Bowers, Newton L. Jr., Hans U. Gerber, James C. Hickman, Donald A. Jones, Cecil J. Nesbitt. *Actuarial Mathematics.* Itasca, Illinois: The Society of Actuaries, 1986.

7. Bowers, Gerber, Hickman, Jones and Nesbitt. *Actuarial Mathematics*.

8. Donate-Armada, Ricardo A. "Cuban Social Security: A Preliminary Actuarial Analysis of Law #24 of Social Security." *Cuba in Transition—Volume 4*. Washington: Association for the Study of the Cuban Economy, 1994.

the Cuban male population, and to model the Cuban social security system. The second mortality table is commonly known as GAM-83 for Males, and it is widely used in the United States for pension valuation. The two mortality tables are reproduced as Table I and Table II, respectively, of the Appendix for ages 25 and older.

The expected future lifetime of the current leadership will depend on its attained age. Both mortality tables are defined for large populations, and therefore they may underestimate the survival of a healthier leadership, or overestimate the longevity of a relatively sicker leadership. One can approximate the survival function of a healthier leadership by assuming that its age is younger than its calendar age. Conversely, the mortality function of a sicker leadership will start an older age than the leadership's calendar age. This is illustrated in Table 1, which shows the expected curtate-future-lifetime for selected ages under the two mortality table assumptions:

Table 1. Curtate-Future-Lifetime for Selected Ages

Age	Cuban Male Mortality	GAM-83 Male Mortality
60	18.3	20.1
65	14.7	16.2
70	11.4	12.7
73[a]	**9.5**	**10.8**
75	8.4	9.6
80	5.9	7.1

a. Age 73 has been highlighted as a benchmark for the current leadership. Fidel Castro Ruz was born on August 13, 1926, and turned 73 years old in 1999.

The GAM-83 mortality table assumes a greater longevity than the Cuban Mortality table. This difference decreases with age as can be observed from Table 1, and Tables I and II in the Appendix.

The economic assumptions of the actuarial model will be defined depending on the type of economic variable one is modeling. For illustrative purposes, this paper will show the result of modeling two types economic variables: annuity payments, and the relative values of macroeconomic amounts. Annuity payments consist of annual payments that will continue as long as the current leadership survives. Examples of annuity payments are the expenditures related to the maintenance of the security apparatus around the leadership, or U.S. federal budget outlays related to countering or influencing the current Cuban leadership.

The relative value of macroeconomic amounts is measured against the absolute value of those amounts at some point in time. An example of this economic variable will be the Cuban gross national product as a multiple of its value in 1999. Another such variable is the GNP lost due to waiting for the leadership's demise before transitioning to a more dynamic, growth-oriented economic system.

ANNUITY PAYMENTS

Annuity payments are annual payments that will continue as long as the leadership survives. The expected total value of these annuity payments can be calculated from formula (3) where f(t) is the accumulated value of those payments at time of death. If the annuity payments are constant over time (i.e. $f(t)=kt$ where k is the first year payments), their expected total value is equal to one year's payment times the expected future lifetime of the leadership. If the annuity payments are assumed to commence on the date of the calculation, i.e. $f(t) = k(t+1)$, then their expected value is equal to one year's payment times the expected curtate-future-lifetime plus one year.

Varying annuity payments can be calculated directly from formula (3) or from the following counterpart of (5):

$$E[f(T_x)] = \sum_{t=0}^{\infty} g(t)\ {}_tp_x \qquad (7)$$

where g(t) is the annuity payments made in year t.

If one assumes that the annuity payments are growing at a fixed rate α (i.e. $g(t) = g(0)\ (1+\alpha)^t$) then:

$$E[f(T_x)] = g(0) \sum_{t=0}^{\infty} (1+\alpha)^t\ {}_tp_x \qquad (8)$$

Based on formula (7) we can define an annuitizing factor as $E[f(T_x)]/g(0)$ which when multiplied by the initial annual payment results in the expected value of all future payments. Tables III and IV of the Ap-

Table 2. Annuitizing Factors for Selected Ages and Expenditures Growth Rates

	Cuban Male Mortality				GAM-83 Male Mortality			
Attained Age	α=0%	α=1%	α=5%	α=10%	α=0%	α=1%	α=5%	α=10%
60	19.319	21.662	36.195	77.957	21.141	23.938	42.042	98.892
65	15.695	17.252	26.217	48.540	17.193	19.077	30.385	61.078
70	12.355	13.331	18.577	30.030	13.682	14.900	21.724	37.924
73*	10.506	11.220	14.913	22.412	11.812	12.731	17.688	28.613
75	9.350	9.921	12.805	18.404	10.654	11.409	15.382	23.739
80	6.882	7.199	8.716	11.373	8.142	8.593	10.835	15.077

pendix show the annuitizing factors for some representative expenditure growth rates for each single age 25 and older. The annuitizing factors increase with the payment growth rate, and they decrease with the age of the leadership. In the absence of any competing goals, anyone benefiting from annuity payments contingent on the survival of the current Cuban leadership benefits from both the increase in the amount of those payments, and the longevity of the leadership.

Table 2 shows the annuitizing factor for some representative growth rates and ages. The annuitizing factors in Table 2 show that the expected accumulated value of annuity payments is greater than the accumulated value of annuity payments paid during the expected curtate-future-lifetime. This is due to the finite probability of surviving the expected curtate-future-lifetime, and receiving payments afterwards.

RELATIVE VALUE OF ECONOMIC VARIABLES

Formula (6) can be used to determine the expected value of an economic variable n years from the date of the calculation. One such economic variable is the relative size of the Cuban Gross National Product at some future date. The relative size of the Cuban GNP n years into the future is equal to the cumulative product of the annual growth factors experienced between the original measurement date and the projected date n years afterwards:

$$GNP(n) = GNP_0 \prod_{k=0}^{n-1} (1 + \gamma_k) \qquad (9)$$

where γ_k is the GNP annual growth rate in year k.

One can estimate the future size of the Cuban GNP using the following formula (10) assuming that the Cuban economy will grow at an average rate γ_o before the current leadership passes away, and an average rate γ_1 afterwards, and that the current leadership passes away t years from now:

$$GNP(t,n) = GNP_0\ (1+ \gamma_0)^t\ (1+ \gamma_1)^{n-t} \qquad (10)$$

Making t the discrete random variable described above, and combining formulas (6) and (10) permits the calculation of the expected value of the Cuban GNP n years from now:

$$E[GNP(T_x,n)] = GNP_0 \left[\sum_{t=0}^{n-1} (1 + \gamma_0)^t\ (1+ \gamma_1)^{n-t}\ {}_tp_x q_{x+t} + (1+ \gamma_0)^n\ {}_np_x \right] \qquad (11)$$

The GNP loss due to waiting for the passing away of the current leadership can be calculated as the difference between the estimated GNP assuming that it always grew at the post-leadership rate and the estimated GNP value calculated using (10). The loss and its expected value can be calculated as follows:

$$Loss(t,n) = GNP_0\ (1+ \gamma_1)^n - GNP_0\ (1+ \gamma_0)^t\ (1+ \gamma_1)^{n-t} \qquad (12)$$

$$E[Loss(T_x,n)] = GNP_0\ (1+ \gamma_1)^n - E[(GNP(T_x,n)] \qquad (13)$$

Dividing (12) by GNP(0,n) scales the GNP loss in units of the GNP projected n years into the future assuming that the post-leadership growth rate starts immediately. The resulting scaled loss does not depend on the actual values of γ_0, and γ_1, but on the ratio of the GNP growth factor while the leadership survives, $(1+ \gamma_0)$, and the GNP growth factor after the leadership dies, $(1+\gamma_1)$ as shown below:

$$\text{Scaled Loss} = \frac{Loss}{GNP_0(1 + \gamma_1)^n} \qquad (14)$$

$$\text{Scaled Loss} = 1 - (1+ \gamma_0)^t\ (1+ \gamma_1)^{-t} \qquad (15)$$

For small γ_0, and γ_1, equivalent scenarios can be defined by the difference $\gamma_1 - \gamma_0$. This simplifies the numerical modeling to calculating the scaled loss for a few scenarios defined by the difference $\gamma_1 - \gamma_0$ rather than modeling each possible combination of the GNP growth rates γ_0, and γ_1.

The Ultimate GNP loss is defined as the expected value of the GNP loss as $n \to \infty$. It is a measurement of the permanent GNP loss because of waiting for the passing away of the current leadership before transitioning to a more growth-oriented economy. Because T_x is a discrete random variable with a probability function that becomes null at a finite future age, the ultimate GNP loss can be easily calculated equal to the expected GNP Loss at the last age with a non-zero survival probability. This last age is age 100 for the Cuban Male Mortality table, and age 110 for the GAM-83 Male Mortality Table. Table 3 shows that Ultimate GNP loss is dependent on the assumed leadership mortality, but that effect is overshadowed by the impact of the difference in GNP growth rates during and after the current leadership.

Table 3. Expected Ultimate Scaled GNP Loss *(Assuming Current Leadership is 73 Years Old)*

$\gamma_1 - \gamma_0$	Cuban Male Mortality	GAM-83 Male Mortality
1.0%	9%	10%
2.0%	17%	19%
3.0%	23%	26%
5.0%	34%	38%
7.5%	45%	49%
10.0%	53%	57%

Table 4 illustrates that the expected Ultimate GNP loss decreases with the assumed age of the leadership. This is due to the increased mortality associated with older ages and the resulting quicker transition to a post-leadership GNP growth scenario. Table 4 also shows that an increase of one year in age results in a decrease of between 1.25% and 1.75% in the ultimate Scaled GNP loss but that is easily overshadowed by an increase of 1% in the difference of GNP growth rates during and after the leadership.

Table 4. Expected Ultimate Scaled GNP Loss as a Function of the Age of the Current Leadership *(Assuming GAM83 Male Mortality)*

Age	$\gamma_1 - \gamma_0 = 3\%$	$\gamma_1 - \gamma_0 = 5\%$	$\gamma_1 - \gamma_0 = 10\%$
60	43%	59%	78%
65	36%	51%	71%
70	30%	43%	62%
73	26%	38%	57%
75	24%	35%	53%
80	18%	27%	43%

As shown in Table 5, the GNP loss grows to its ultimate value relatively fast reaching 50% of the ultimate level in 4 to 5 years, and 90% of its ultimate value within 13 years. The size of the difference in GNP growth rates during and after the current leadership accelerates the rate at which the expected GNP loss reaches its ultimate value.

Table 5. Expected Scaled GNP Loss as a Function of Elapsed Years *(Assuming GAM83 Male Mortality and Current Leadership is 73 Years Old)*

Year	$\gamma_1 - \gamma_0 = 3\%$	$\gamma_1 - \gamma_0 = 5\%$	$\gamma_1 - \gamma_0 = 10\%$
0	0%	0%	0%
1	3%	5%	9%
2	5%	9%	16%
3	8%	13%	23%
4	10%	16%	29%
5	12%	19%	34%
6	14%	22%	38%
7	16%	24%	41%
8	17%	27%	44%
9	19%	28%	47%
10	20%	30%	49%
11	21%	32%	51%
12	22%	33%	52%
13	23%	34%	53%
14	23%	35%	54%
Ultimate	26%	38%	57%

All of these illustrations assume that the transition to a more dynamic growth-oriented economy will commence immediately after the current leadership passes away. The expected GNP loss will increase if a delay is introduced in the start of the post-leadership growth rates. Conversely, if one assumes that the transition to the higher GNP growth scenario commences before the leadership passes away, the expected GNP loss will be reduced.

FINAL COMMENTS

An actuarial model of the impact of waiting for the passing of the current Cuban leadership will be highly dependent on the survival function assumed for the leadership. This paper has limited itself to modeling one possible political transition scenario: no change will occur until the current leadership dies. One can argue that the current leadership is subject to risks that may warrant the use of a more aggressive mortality table. Nonetheless in the absence of any empirical evidence supporting this assertion, a general population mortality table appears to be the most appropriate assumption for the survival function of the current leadership.

The illustrative examples in this paper were based on individual survival models. Collective survival functions can also serve as the basis for an actuarial model. Collective survival functions can be established such that the current leadership is considered to have survived as long as one member of the collective is alive, or if the members of the collective pass away in a certain chronological order. These survival models can be built based on individual mortality tables and the demographic characteristics of the leadership. Assuming that the leadership will survive as long as any member is alive will have the result of increasing the longevity of the leadership, and the relative size of the annuitizing factors, and the Ultimate GNP loss.

The expected value of the ultimate GNP loss can be used to support an argument for the immediate transition to a market economy even if this transition is likely to result in some short term contraction of the economy. As long as the size of that immediate contraction does not exceed the expected Ultimate GNP loss, it can be argued that the Cuban economy is better off in the long term. The only justifications for maintaining the current leadership under these circumstances can be found in political and social arguments, which are outside of the scope of this paper.

A thorough discussion of the gap between the GNP growth rates under the current leadership, and the post-leadership GNP growth rates is also outside the scope of this paper. It should be pointed out that the growth rates could be assumed to be random variables. Such a model requires the establishment of probability distribution for the GNP growth rates during and after the current leadership.

An actuarial model can serve as a powerful tool for quantifying the effect of survival of individuals, organisms, objects, and processes. In the case of the Cuban economy, it has been applied to show the likely impact on the Cuban economy of waiting for the current leadership to die. The model presented here shows that waiting for the current leadership to pass away before transitioning to a more dynamic, growth-oriented economy will likely result in lasting losses to the Cuban economy that will be realized during the expected lifetime of the current leadership.

APPENDIX

Table I. Selected Values from Cuban Male Mortality Table for Ages 25 and Higher

Age x	q_x	e_x	Age x	q_x	e_x
25	0.001626	48.9	70	0.031457	11.4
26	0.001657	47.9	71	0.034002	10.7
27	0.001479	47.0	72	0.038567	10.1
28	0.001723	46.1	73	0.041667	9.5
29	0.001978	45.2	74	0.046059	8.9
30	0.001794	44.3	75	0.052916	8.4
31	0.002060	43.3	76	0.057078	7.8
32	0.001871	42.4	77	0.063932	7.3
33	0.002151	41.5	78	0.068966	6.8
34	0.002198	40.6	79	0.086025	6.3
35	0.002248	39.7	80	0.094400	5.9
36	0.002301	38.8	81	0.103475	5.5
37	0.002355	37.9	82	0.113013	5.1
38	0.002680	37.0	83	0.123025	4.8
39	0.002473	36.1	84	0.134063	4.5
40	0.002817	35.2	85	0.146400	4.1
41	0.003178	34.3	86	0.158963	3.9
42	0.003262	33.4	87	0.171350	3.6
43	0.003351	32.5	88	0.184100	3.3
44	0.003756	31.6	89	0.198350	3.1
45	0.003862	30.7	90	0.214613	2.8
46	0.004305	29.8	91	0.232125	2.6
47	0.004772	28.9	92	0.250288	2.4
48	0.004916	28.1	93	0.268688	2.2
49	0.005429	27.2	94	0.287200	2.0
50	0.005972	26.4	95	0.304225	1.8
51	0.006549	25.5	96	0.320463	1.6
52	0.006765	24.7	97	0.335850	1.4
53	0.007812	23.9	98	0.350375	1.1
54	0.008082	23.1	99	0.350375	0.6
55	0.008807	22.2	100	1.000000	0.0
56	0.009580	21.4			
57	0.009938	20.6			
58	0.011297	19.9			
59	0.012245	19.1			
60	0.013786	18.3			
61	0.014349	17.6			
62	0.016092	16.8			
63	0.017964	16.1			
64	0.018750	15.4			
65	0.020888	14.7			
66	0.022526	14.0			
67	0.024303	13.3			
68	0.026237	12.7			
69	0.028346	12.0			

Table II. Selected Values from GAM83-Male Mortality Table for Ages 25 and Higher

Age x	q_x	e_x	Age x	q_x	e_x
25	0.000464	52.5	70	0.027530	12.7
26	0.000488	51.5	71	0.030354	12.0
27	0.000513	50.5	72	0.033370	11.4
28	0.000542	49.6	73	0.036680	10.8
29	0.000572	48.6	74	0.040388	10.2
30	0.000607	47.6	75	0.044597	9.7
31	0.000645	46.6	76	0.049388	9.1
32	0.000687	45.7	77	0.054758	8.6
33	0.000734	44.7	78	0.060678	8.1
34	0.000785	43.7	79	0.067125	7.6
35	0.000860	42.8	80	0.074070	7.1
36	0.000907	41.8	81	0.081484	6.7
37	0.000966	40.8	82	0.089320	6.3
38	0.001039	39.9	83	0.097525	5.9
39	0.001128	38.9	84	0.106047	5.6
40	0.001238	38.0	85	0.114836	5.2
41	0.001370	37.0	86	0.124170	4.9
42	0.001527	36.1	87	0.133870	4.6
43	0.001715	35.1	88	0.144073	4.3
44	0.001932	34.2	89	0.154859	4.0
45	0.002183	33.2	90	0.166307	3.8
46	0.002471	32.3	91	0.178214	3.5
47	0.002790	31.4	92	0.190460	3.3
48	0.003138	30.5	93	0.203007	3.1
49	0.003513	29.6	94	0.217904	2.9
50	0.003909	28.7	95	0.234086	2.7
51	0.004324	27.8	96	0.248436	2.5
52	0.004755	26.9	97	0.263954	2.3
53	0.005200	26.0	98	0.280803	2.1
54	0.005660	25.2	99	0.299154	1.9
55	0.006131	24.3	100	0.319185	1.8
56	0.006618	23.5	101	0.341086	1.6
57	0.007139	22.6	102	0.365052	1.4
58	0.007719	21.8	103	0.393102	1.3
59	0.008384	21.0	104	0.427255	1.1
60	0.009158	20.1	105	0.469531	0.9
61	0.010064	19.3	106	0.521945	0.7
62	0.011133	18.5	107	0.586518	0.6
63	0.012391	17.7	108	0.665268	0.3
64	0.013868	17.0	109	1.000000	0.0
65	0.015592	16.2			
66	0.017579	15.4			
67	0.019804	14.7			
68	0.022229	14.0			
69	0.024817	13.3			

Table III. Annuitizing Factors for Ages 25 and Higher for Select Growth Rates Assuming Cuban Male Mortality

Age x	α=0%	α=1%	α=5%	α=10%	Age x	α=0%	α=1%	α=5%	α=10%
25	49.870	65.773	254.966	2,120.253	70	12.355	13.331	18.577	30.030
26	48.949	64.236	242.266	1,929.731	71	11.723	12.605	17.284	27.248
27	48.029	62.714	230.159	1,756.302	72	11.101	11.895	16.054	24.702
28	47.099	61.193	218.569	1,598.093	73	10.506	11.220	14.913	22.412
29	46.178	59.700	207.567	1,454.408	74	9.919	10.558	13.826	20.312
30	45.268	58.234	197.120	1,323.899	75	9.350	9.921	12.805	18.404
31	44.347	56.769	187.117	1,204.797	76	8.817	9.326	11.871	16.705
32	43.437	55.331	177.620	1,096.620	77	8.290	8.742	10.980	15.142
33	42.516	53.894	168.525	997.885	78	7.788	8.189	10.154	13.734
34	41.606	52.483	159.891	908.213	79	7.290	7.645	9.364	12.434
35	40.695	51.086	151.658	826.556	80	6.882	7.199	8.716	11.373
36	39.785	49.701	143.807	752.196	81	6.496	6.777	8.114	10.413
37	38.874	48.330	136.321	684.480	82	6.130	6.380	7.557	9.545
38	37.964	46.972	129.181	622.813	83	5.784	6.006	7.041	8.758
39	37.063	45.640	122.405	566.803	84	5.455	5.651	6.560	8.042
40	36.152	44.307	115.911	515.642	85	5.144	5.318	6.115	7.393
41	35.252	43.000	109.748	469.178	86	4.855	5.009	5.707	6.809
42	34.361	41.716	103.900	426.973	87	4.584	4.719	5.330	6.279
43	33.470	40.445	98.320	388.516	88	4.325	4.444	4.977	5.791
44	32.579	39.186	92.998	353.471	89	4.075	4.179	4.642	5.338
45	31.698	37.950	87.947	321.637	90	3.836	3.927	4.327	4.920
46	30.817	36.726	83.128	292.618	91	3.611	3.690	4.034	4.537
47	29.946	35.526	78.555	266.253	92	3.400	3.468	3.764	4.188
48	29.085	34.348	74.216	242.296	93	3.201	3.260	3.511	3.865
49	28.224	33.181	70.074	220.443	94	3.009	3.059	3.270	3.562
50	27.372	32.036	66.144	200.583	95	2.819	2.860	3.032	3.267
51	26.531	30.913	62.415	182.529	96	2.614	2.647	2.782	2.963
52	25.699	29.812	58.876	166.114	97	2.376	2.400	2.497	2.626
53	24.867	28.721	55.495	151.126	98	2.072	2.087	2.147	2.225
54	24.055	27.663	52.309	137.553	99	1.650	1.656	1.682	1.715
55	23.243	26.614	49.264	125.151	100	1.000	1.000	1.000	1.000
56	22.441	25.586	46.374	113.867					
57	21.648	24.578	43.631	103.599					
58	20.855	23.579	41.009	94.208					
59	20.082	22.611	38.539	85.703					
60	19.319	21.662	36.195	77.957					
61	18.575	20.743	33.987	70.939					
62	17.831	19.832	31.874	64.506					
63	17.106	18.951	29.884	58.677					
64	16.401	18.098	28.012	53.393					
65	15.695	17.252	26.217	48.540					
66	15.008	16.435	24.529	44.140					
67	14.331	15.634	22.925	40.122					
68	13.663	14.850	21.401	36.452					
69	13.004	14.082	19.953	33.097					

Table IV. Annuitizing Factors for Ages 25 and Higher for Select Growth Rates Assuming GAM83-Male Mortality

Age x	α=0%	α=1%	α=5%	α=10%	Age x	α=0%	α=1%	α=5%	α=10%
25	53.476	71.533	298.648	2,792.049	70	13.682	14.900	21.724	37.924
26	52.500	69.868	283.606	2,538.495	71	13.041	14.152	20.296	34.517
27	51.525	68.219	269.280	2,307.940	72	12.418	13.430	18.952	31.424
28	50.551	66.588	255.636	2,098.294	73	11.812	12.731	17.688	28.613
29	49.578	64.973	242.642	1,907.665	74	11.223	12.057	16.498	26.059
30	48.606	63.376	230.267	1,734.324	75	10.654	11.409	15.382	23.739
31	47.635	61.796	218.482	1,576.706	76	10.104	10.787	14.336	21.637
32	46.665	60.233	207.259	1,433.385	77	9.577	10.193	13.361	19.736
33	45.696	58.687	196.573	1,303.063	78	9.074	9.630	12.454	18.019
34	44.729	57.158	186.396	1,184.563	79	8.596	9.096	11.613	16.472
35	43.763	55.645	176.707	1,076.812	80	8.142	8.593	10.835	15.077
36	42.800	54.151	167.484	978.853	81	7.714	8.119	10.116	13.821
37	41.838	52.673	158.700	889.764	82	7.309	7.674	9.452	12.689
38	40.878	51.210	150.336	808.748	83	6.928	7.256	8.840	11.669
39	39.919	49.765	142.372	735.081	84	6.569	6.863	8.273	10.747
40	38.963	48.337	134.792	668.100	85	6.229	6.494	7.748	9.912
41	38.010	46.926	127.579	607.206	86	5.908	6.145	7.261	9.153
42	37.061	45.534	120.717	551.852	87	5.604	5.816	6.808	8.463
43	36.116	44.160	114.191	501.541	88	5.315	5.505	6.386	7.833
44	35.176	42.806	107.986	455.819	89	5.041	5.212	5.993	7.257
45	34.243	41.473	102.088	414.272	90	4.782	4.934	5.627	6.731
46	33.315	40.159	96.485	376.524	91	4.536	4.672	5.286	6.249
47	32.395	38.868	91.164	342.231	92	4.303	4.424	4.967	5.807
48	31.483	37.598	86.110	311.078	93	4.080	4.188	4.667	5.398
49	30.579	36.349	81.313	282.776	94	3.865	3.960	4.382	5.017
50	29.683	35.123	76.758	257.063	95	3.663	3.747	4.118	4.669
51	28.796	33.918	72.434	233.698	96	3.477	3.552	3.877	4.355
52	27.917	32.733	68.327	212.463	97	3.296	3.361	3.646	4.058
53	27.045	31.569	64.428	193.157	98	3.119	3.177	3.423	3.777
54	26.181	30.425	60.723	175.601	99	2.947	2.996	3.209	3.510
55	25.325	29.299	57.203	159.632	100	2.778	2.820	3.002	3.256
56	24.475	28.192	53.857	145.101	101	2.611	2.647	2.801	3.012
57	23.631	27.102	50.675	131.873	102	2.445	2.475	2.603	2.776
58	22.794	26.029	47.650	119.831	103	2.276	2.301	2.404	2.543
59	21.964	24.974	44.774	108.869	104	2.102	2.122	2.203	2.311
60	21.141	23.938	42.042	98.892	105	1.924	1.939	2.001	2.081
61	20.327	22.920	39.449	89.815	106	1.742	1.753	1.796	1.853
62	19.523	21.924	36.990	81.562	107	1.552	1.559	1.587	1.622
63	18.732	20.950	34.662	74.062	108	1.335	1.338	1.351	1.368
64	17.954	20.000	32.462	67.254	109	1.000	1.000	1.000	1.000
65	17.193	19.077	30.385	61.078					
66	16.449	18.181	28.429	55.481					
67	15.726	17.316	26.590	50.415					
68	15.023	16.480	24.864	45.830					
69	14.342	15.676	23.244	41.681					