

# A NOTE ON THE ALGEBRA OF MULTIPLE EXCHANGE RATES<sup>1</sup>

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Underlying the existence of multiple exchange rates is the operation of segmented markets. The Central Bank classifies transactions with nonresidents in different categories, each using a different exchange rate. Administrative controls of foreign exchange reserves rule out the pursuit of arbitrage opportunities. The operation of multiple exchange rates requires from economic agents the use of different “accounts” for each exchange rate. Multiple exchange rates were common in the 1960s and 1970s, but their popularity has decreased significantly since the 1990s. The use of a dual currency, dual exchange rate system in Cuba is an example of a multiple exchange rate system.

This paper shows how Cuba’s exchange rate system boils down algebraically to a simple monetary rule, in which the central bank picks (i) the parity between the two domestic legal currencies and (ii) the parity between the convertible domestic currency and foreign currencies, to ensure that foreign exchange reserves are not depleted against the backdrop of a sys-

tematic excess supply in monetary base denominated in Cuban Pesos (CUPs). This note builds on Di Bella and Romeu (2017), and the references therein.

## THE ACCOUNTING OF MULTIPLE EXCHANGE RATES

The operation of the Cuban dual currency, dual exchange rate system involves the use of Cuban Pesos (CUPs) in domestic markets where rationing and queues constitute the main adjustment mechanism; and the use of Convertible Cuban Pesos (CUCs) in markets that are less regulated, and generally less rationed (e.g., in the retail outlet system). To understand how different markets interact, we first consolidate all economic sectors’ CUC accounts and the balance of payments, whose transactions are denominated in foreign exchange (FX); the rationale for consolidating FX and CUC accounts is that FX can be converted to CUCs, but not to CUPs at official exchange rates:

$$\begin{aligned} \sum_x \bar{P}_t^x \left[ \sum_j \bar{x}_t^{j,d} - \bar{x}_t^s + \bar{x}_t^{E,d} \right] + \bar{w}_t \left[ \sum_j \bar{L}_t^{j,d} - \bar{L}_t^s \right] = \\ = e_t [(\Delta D_t^{*,s} - \Delta D_t^{*,d}) + (\Delta R_t^{*,s} - \Delta R_t^{*,d})] + [\Delta Bc_t^s - \Delta Bc_t^d] \end{aligned} \quad (1)$$

In expression (1), quantities and prices denominated in CUCs are denoted with an overline bar. The first term on the left-hand side (LHS) denotes all goods

markets (indexed by  $x$ ), which transact in CUCs. Good markets transacting in CUCs include generally tradable goods, but they could include non-tradable

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2. The views expressed in this paper are those of the authors and do not necessarily represent those of the IMF or IMF policy.

goods as well. Intersectoral domestic flows (e.g., interest payments, transfers or tax payments) are consolidated out. The supra-index  $s$  denotes supply,  $d$  denotes demand, while all domestic sectors are indexed by  $j$  (including the consolidated government, firms, and households); the supra-index  $E$  denotes the external sector (and thus this sector's demand are the country's exports). The second term in the LHS denotes the labor market operating in CUCs. The terms on the right-hand side (RHS) are financial

$$\begin{aligned} \sum_x P_t^x \left[ \sum_j x_t^{j,d} - x_t^s + x_t^{E,d} \right] + w_t \left[ \sum_j L_t^{j,d} - L_t^s \right] + \bar{e}_t \sum_x \bar{P}_t^x \bar{x}_t^{HH,CUP} = \\ = [\Delta D_t^s - \Delta D_t^d] + [\Delta B_t^s - \Delta B_t^d] \end{aligned} \quad (2)$$

In expression (2), the interpretation of the terms is analogous to that in expression (1). The first two terms of the LHS denote goods and labor markets operating in CUPs, while the third term is the demand by households ( $HH$ ) of goods transacted in CUC markets, by previously converting their CUP flows into CUCs at the exchange rate between the two domestic currencies  $\bar{e}_t$ . Goods markets that transact in CUPs include both tradable and non-tradable goods in the rationing system. The RHS shows domestic debt and monetary base markets in CUPs, respectively.

Combining each sector's budget constraint in both CUPs and CUCs into expressions (1) and (2) allows

$$\begin{aligned} \sum_x P_t^x \left[ \sum_j x_t^{j,d} - x_t^s + x_t^{E,d} \right] + w_t \left[ \sum_j L_t^{j,d} - L_t^s \right] \\ + \bar{e}_t \sum_x \bar{P}_t^x \left[ \sum_j \bar{x}_t^{j,d} - \bar{x}_t^s + \bar{x}_t^{E,d} \right] + \bar{e}_t \bar{w}_t \left[ \sum_j \bar{L}_t^{j,d} - \bar{L}_t^s \right] = \\ = [\Delta D_t^s - \Delta D_t^d] + [\Delta B_t^s - \Delta B_t^d] \\ + \bar{e}_t \{ e_t [(\Delta D_t^{*,s} - \Delta D_t^{*,d}) + (\Delta R_t^{*,s} - \Delta R_t^{*,d})] + [\Delta B C_t^s - \Delta B C_t^d] \} \end{aligned} \quad (3)$$

Expression (3) underlines the familiar results that (i) if  $n - 1$  markets are in equilibrium, market  $n$  will also be in equilibrium; and (ii) the summation of *ex-ante*

flows. The CUC/USD exchange rate is denoted by  $e_t$ ;  $D^*$  and  $R^*$  are external debt and foreign exchange reserves, respectively (both denominated in foreign exchange), and  $\Delta$  is the difference operator.<sup>3</sup> Finally,  $Bc$  denotes the monetary base in CUCs.

We then consolidate the CUP accounts of all economic sectors, which allows to see all markets transacting in CUPs:

to identify the economy's "markets": rationed goods markets operating in CUPs (including both tradable and non-tradable goods); non-rationed (or less rationed) goods market operating in CUCs; the labor market (differentiating workers paid in CUPs or CUCs); the monetary base markets (in CUPs or CUCs); the CUP-denominated domestic debt market; and the FX-denominated external debt market.

To understand the interconnection of all markets through the sectors' budget constraints ("Walras Law"), we combine expressions (1) and (2), by multiplying the former by  $\bar{e}_t$ :

disequilibria in all markets must add up to zero. Both results have an immediate interpretation in our context. First, if CUP monetary base is in equilibrium,

3. In the case of Cuba, external debt flows are mainly bilateral and official.

there will not be a disequilibrium in CUC markets originated in CUPs. Second, if there is an *ex-ante* disequilibrium in a market that transacts in CUPs, there must be disequilibria in other markets (including in CUCs), of equal magnitude and opposite sign. In the case of Cuba, given multiple fixed exchange rates, and the inconvertibility of the CUP into foreign cur-

rency, an excess supply in CUPs will be converted into CUCs at the  $\bar{e}_t$  exchange rate, which in turn will be used either for CUC hoarding, or to purchase goods in markets transacting in CUCs. If as a result there is an excess supply of CUC monetary base, it will be converted into FX at the  $e_t$  exchange rate.

### THE FX CONSTRAINT AND MONETARY ADJUSTMENT

Given the fixed exchange rate parities between the CUC and the CUP, and between the CUC and the USD (and assuming away any disequilibria in debt markets, which most likely represent official external transactions or non-voluntary domestic debt), any *ex-ante* disequilibrium ends up being reflected in the monetary base market (in CUPs, CUCs, or both). In other words, rationed goods markets (both non-tradable and tradable goods) adjust by quantities, with

the price mechanism playing a minor role. Non-rationed goods markets (which we assume are only traded goods) adjust through net external exports (imports), whose counterpart is a change in foreign reserves of the Central Bank. Any disequilibrium in goods markets has a counterpart disequilibrium in monetary base markets, which adjust through net foreign exchange purchases (or sales) at the official exchange rates:

$$[\Delta B_t^s - \Delta B_t^{d,HH}] + \bar{e}_t[\Delta Bc_t^s - \Delta Bc_t^{d,HH}] = -\bar{e}_t e_t \Delta R_t^{E,S} \quad (4)$$

Expression (4) shows that any disequilibrium in the monetary base market, either in CUPs (the first term on the LHS), or CUCs (the second term on the LHS), will end up being resolved with a change in foreign reserves (the term on the RHS). In particular, a large excess supply in CUP monetary base adjusts through purchases of CUC monetary base, which, if not hoarded, result in a deficit in the country's balance of payments that is financed through a decrease in the Central Bank international reserves. The LHS

can also be interpreted as the country's (negative) foreign exchange demand.

In other words, CUP (CUC) monetary base supply reflects the deficit in the CUP (CUC) account of the general government, net of CUP (CUC/FX) debt issuance. Any excess supply (demand) in monetary base is reflected in a loss (increase) in the international reserves of the central bank. Such a change in reserves is identical to the country's overall balance of payments:

$$TB_t^*(\rho) + Gr_t^* + Re_t^* - D_{t-1}^*(1 + i_{t-1}^*) + D_t^* = \Delta R_t^{E,S} \quad (5)$$

In expression (5),  $TB^*$  is the trade balance,  $Gr^*$  denotes the grants,  $RE^*$  are remittances (all expressed in foreign currency), and  $i^*$  is the interest rate paid on external debt. Note that the trade balance is shown to depend on  $\rho$ , which we take as the share of tradable goods that are transacted in rationed markets ( $0 < \rho < 1$ ). We assume that the higher this share, the lower

the trade balance surplus, other things equal. This turns out to be the case, as the adjustment mechanism in this market is supply rationing, and any unsatisfied demand would be spilled over CUC markets at much higher CUP prices. Taking this into consideration, expression (4) can be re-expressed as:

$$\begin{aligned} & [(Def_t^G - \Delta D_t) - \Delta B_t^{d,HH}] + \bar{e}_t [(\overline{Def}_t^G - e_t \Delta D_t^*) - \Delta Bc_t^{d,HH}] = \\ & = -\bar{e}_t e_t [TB_t^*(\rho) + Gr_t^* + Re_t^* - D_{t-1}^*(1 + i_{t-1}^*) + D_t^*] \end{aligned} \quad (6)$$

In expression (6), the first term on the LHS will be larger, the larger is  $\rho$ , while the second term on the LHS will be lower. In other words, the more binding the external constraint (the term on the RHS), the smaller will be the share of goods that are transacted in rationed markets. A sufficiently depreciated CUP *vis-a-vis* the CUC ensures that whenever gross foreign exchange inflows are very reduced, a decrease in the share of tradable goods supply in the rationed markets (i.e., a decrease in the per capita allocation of rationed goods) will result in a relatively modest disequilibrium in the balance of payments, and thus, in an also moderate international reserve loss.

Consider, for instance, oil and derivatives supply in domestic markets (clearly an imported good for Cuba): The lower gross FX inflows, the lower energy imports, the lower domestic output, the higher hidden unemployment and the excess supply of CUP monetary base. The excess CUPs end up being sterilized in exchange for CUCs. Parallel exchange rate markets arise whenever the CUC convertibility (either to CUPs or to FX) is suspended, even partially. A more depreciated CUP/CUC exchange rate is needed whenever gross FX inflows are very low (e.g., in the 1990s). The same type of adjustment could be implemented if multiple exchange rates would be maintained, but currency would be unified. To see this, consider a Central Bank that administratively changes the amount of foreign exchange available to import a commodity that could be used for different purposes by many economic sectors: if the Central Bank operating a multiple exchange rate system increases the foreign exchange available to import such a commodity at a more depreciated rate (*vis-a-vis*, e.g., the USD) and decreases it at a more appreciated exchange rate, it would be *de facto* depreciating the currency, without the need to change any of the existing multiple exchange rate parities. This fact highlights that the problem of currency unification in Cuba is one of multiple exchange rates, as highlighted in Di Bella and Romeu (2017). The inconvertibility of the CUP, and the use of the CUC as a bridge to exchange CUPs into FX through a very depreciated exchange rate (24 CUPs/CUC), makes the system relatively easier to operate, with the CUP excess sup-

ply being sterilized at a relatively small cost in terms of FX.

## FINAL REMARKS

This paper discusses the algebra of multiple exchange rates (which is valid for any country with a multiple exchange rate system), and applies it to Cuba, a country with a dual currency, dual exchange rate system. It then derives the economy's "Walras Law" by consolidating all sectors' CUP accounts and CUC accounts, using appropriate exchange rates.

The paper highlights that the CUP is transacted in markets with rationed supply, while the CUC is used in markets that are less rationed. As a straightforward application of Walras' Law, the paper shows how changes in international reserves are the counterpart of *ex-ante* excess supply in the monetary base (both in CUP and CUC). The paper also shows that the degree to which tradable goods are rationed depends on how binding the foreign exchange constraint is: the more binding it is, the lower the share of the supply of tradable goods that is sold in rationed markets. A change in the share of rationing changes the relative sizes of CUP/CUC monetary base supply (concretely, an increase in the share of tradable goods rationing increases the excess supply in the CUP monetary base). This results in a simple adjustment mechanism that is based on the very depreciated parity of the CUP *vis-a-vis* the CUC, without the need to change any nominal CUP or CUC prices. Such a depreciated parity results in a relatively moderate reserve loss whenever the foreign exchange constraint binds more.

As discussed in Di Bella and Romeu (2017), the simplest of a sustainable currency unification requires only picking unification exchange rate parities such that the consolidated government account (measured in CUPs at unified rates) remains balanced or in surplus. Unification could proceed even keeping a system of multiple exchange rates, what underlines the fact that the dual currency is really not the problem to solve, but rather the existence of multiple exchange rates and that of a non-convertible currency (the CUP) that is in systematic excess supply, both of which are the manifestation of rationing, segmented markets and distorted relative prices. If unification

exchange rates result in a deficit, then currency unification would need to be coupled with a program of

fiscal consolidation, among other policy measures (Di Bella and Wolfe, 2008).

## REFERENCES

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